UNIVERSITÉ Concordia UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
Department of Computer Science and Software Engineering

COMP371 COMPUTER GRAPHICS

LECTURE 16
RAY TRACING/GEOMETRIC QUERIES

Lecture Overview

- Review of last class
- Ray Tracing/Geometric Queries

Ray-Surface Intersection

- Necessary in ray tracing
- General implicit surfaces
- General parametric surfaces
- Specialized analysis for special surfaces
  - Spheres
  - Planes
  - Polygons
  - Quadrics

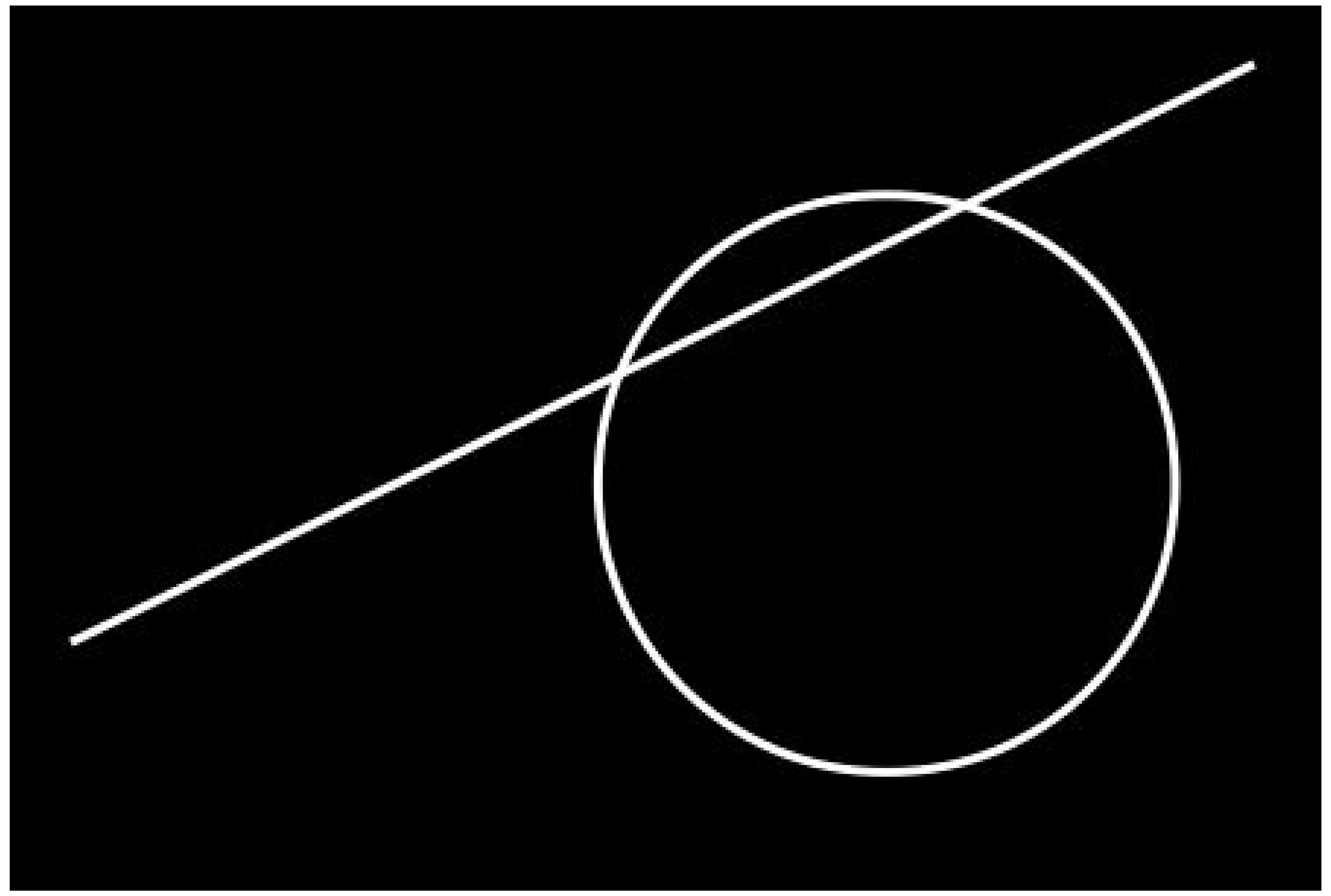
Intersection of Rays and Parametric Surfaces

- Ray in parametric form
  - Origin $p_0 = [x_0\ y_0\ z_0]^T$
  - Direction $d = [x_d\ y_d\ z_d]^T$
  - Assume d is normalized ($x_d^2 + y_d^2 + z_d^2 = 1$)
  - Ray $p(t) = p_0 + dt$ for $t > 0$
  - <u>Surface in parametric form</u>
    - Point $q = g(u,v)$, possible bounds on u,v
    - Solve $p + dt = g(u,v)$
    - Three equations in three unknowns (t, u, v)

Intersection of Rays and Implicit Surfaces

- Ray in parametric form
  - Origin $p_0 = [x_0\ y_0\ z_0]^T$
  - Direction $d = [x_d\ y_d\ z_d]^T$
  - Assume d is normalized ($x_d^2 + y_d^2 + z_d^2 = 1$)
  - Ray $p(t) = p_0 + dt$ for $t > 0$
  - Implicit surface
    - Given by $f(q) = 0$
    - Consists of all points q such that $f(q) = 0$
    - Substitute ray equation for q: $f(p_0 + dt) = 0$
    - Solve for t (univariate root finding)
    - Closed form (if possible), otherwise numerical approximation

Ray-Sphere Intersection I



- Common and easy case
- Define sphere by
  - Center c = $[x_c\ y_c\ z_c]^T$
  - Radius r
  - Surface $f(q) = (x-x_c)^2 + (y-y_c)^2 + (z-z_c)^2 - r^2 = 0$
- Plug in ray equations for x, y, z:

$$x = x_0 + x_d t, \qquad y = y_0 + y_d t, \qquad z = z_0 + z_d t$$

- And we obtain a scalar equation for t:

$$(x_0 + x_d t - x_c)^2 + (y_0 + y_d t - y_c)^2 + (z_0 + z_d t - z_c)^2 = r^2$$

Ray-Sphere Intersection II

Simplify to $at^2 + bt + c = 0$

where

$$a = x_d^2 + y_d^2 + z_d^2 = 1 \qquad \text{since } |d| = 1$$

$$b = 2(x_d(x_0 - x_c) + y_d(y_0 - y_c) + z_d(z_0 - z_c))$$

$$c = (x_0 - x_c)^2 + (y_0 - y_c)^2 + (z_0 - z_c)^2 - r^2$$

Solve to obtain t_0 and t_1

$$t_{0,1} = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$

Check if t_0 , t_1 > 0 (ray)
Return min(t_0 , t_1)

Ray-Sphere Intersection III

For lighting, calculate unit normal

$$n = \frac{1}{r}[(x_i - x_c) \quad (y_i - y_c) \quad (z_i - z_c)]^T$$

Negate if ray originates inside the sphere

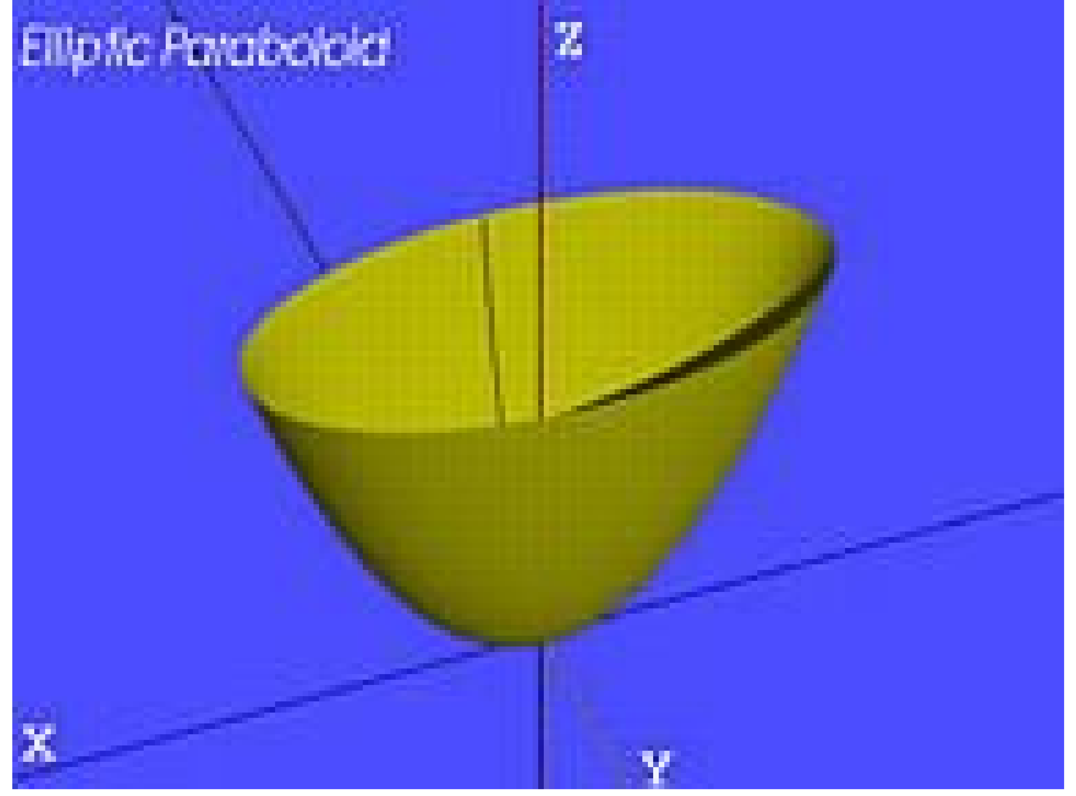
Note possible problems with roundoff errors

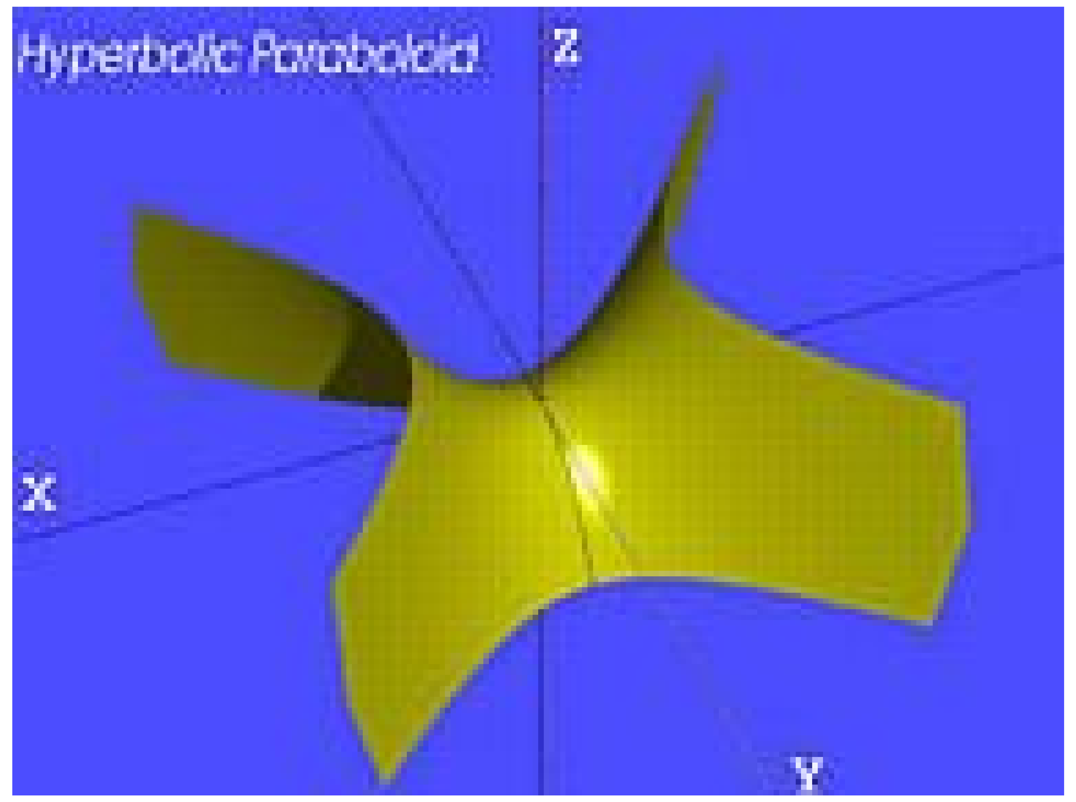
Simple Optimizations

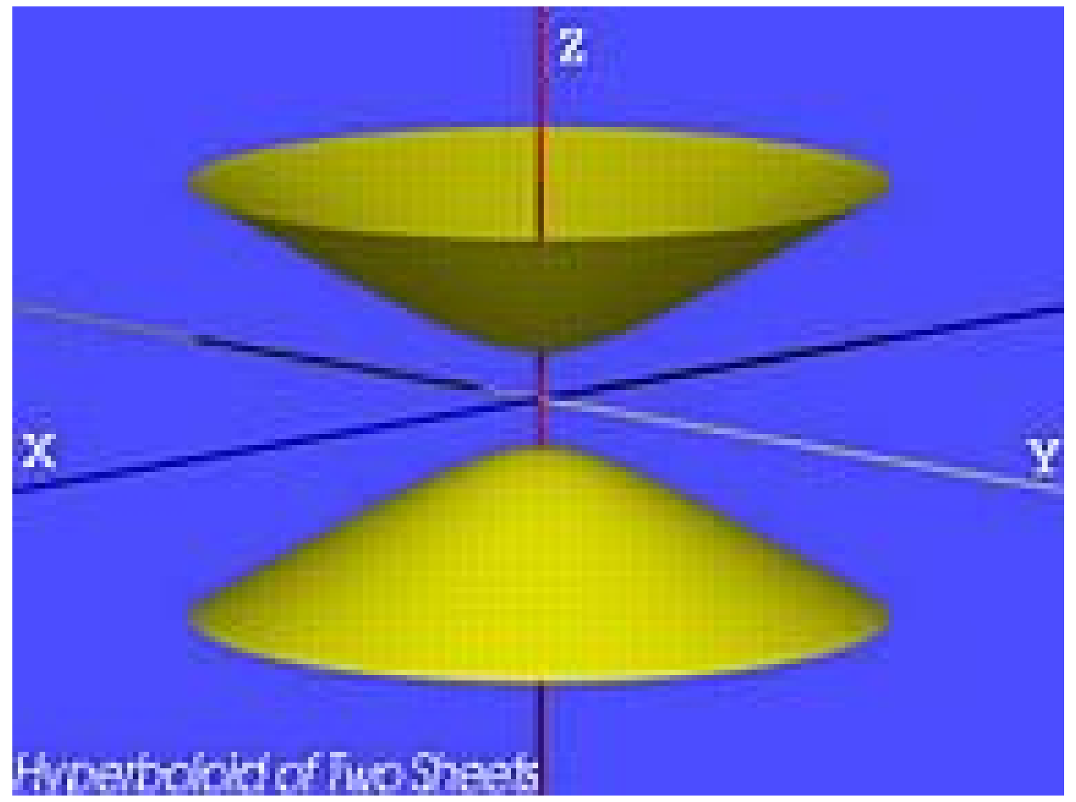
- Factor common subexpressions
- Compute only what is necessary
  - Calculate $b^2 - 4c$, abort if negative
  - Compute normal only for closest intersection
  - Other similar optimizations

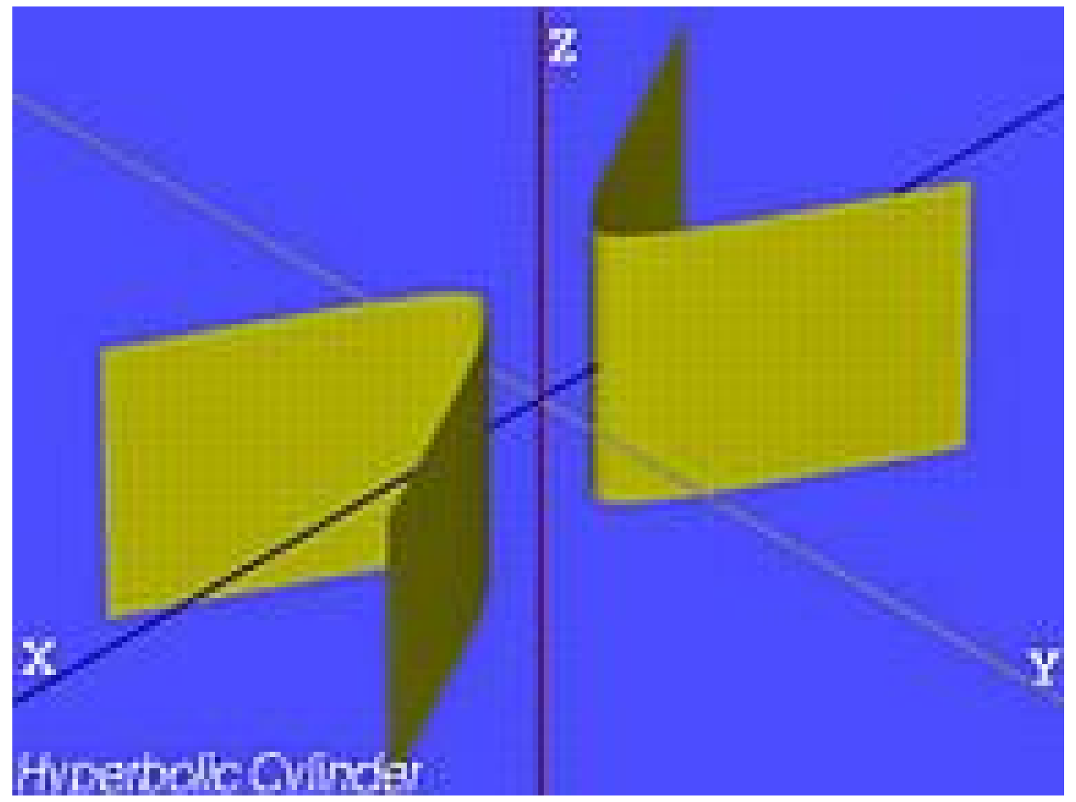
Ray-Quadric Intersection

- Quadric f(p) = f(x,y,z) = 0,
  where f is polynomial of order 2
- Sphere, ellipsoid, paraboloid, hyperboloid, cone, cylinder
- Closed form solution as for sphere
- Important case for modelling in ray tracing
- Combine with CSG









Ray-Polygon Intersection I

- Assume planar polygon in 3D
  - Intersect ray with plane containing polygon
  - Check if intersection point is inside polygon
- Plane
  - Implicit form: ax + by + cz + d = 0
  - Unit normal: $n = [a\ b\ c]^T$ with $a^2 + b^2 + c^2 = 1$
- Substitute $a(x_0 + x_d t) + b(y_0 + y_d t) + c(z_0 + z_d t) + d = 0$
- Solve

$$t = \frac{-(ax_0 + by_0 + cz_0 + d)}{ax_d + by_d + cz_d}$$

Ray-Polygon Intersection II

- Substitute t to obtain intersection point in plane
- Rewrite using dot product

$$t = \frac{-(ax_0 + by_0 + cz_0 + d)}{ax_d + by_d + cz_d} = \frac{-(n \cdot p_0 + d)}{n \cdot d}$$

- If n.d = 0, no intersection (ray parallel to plane)
- if t <= 0, the intersection is behind ray origin

Test if point inside polygon

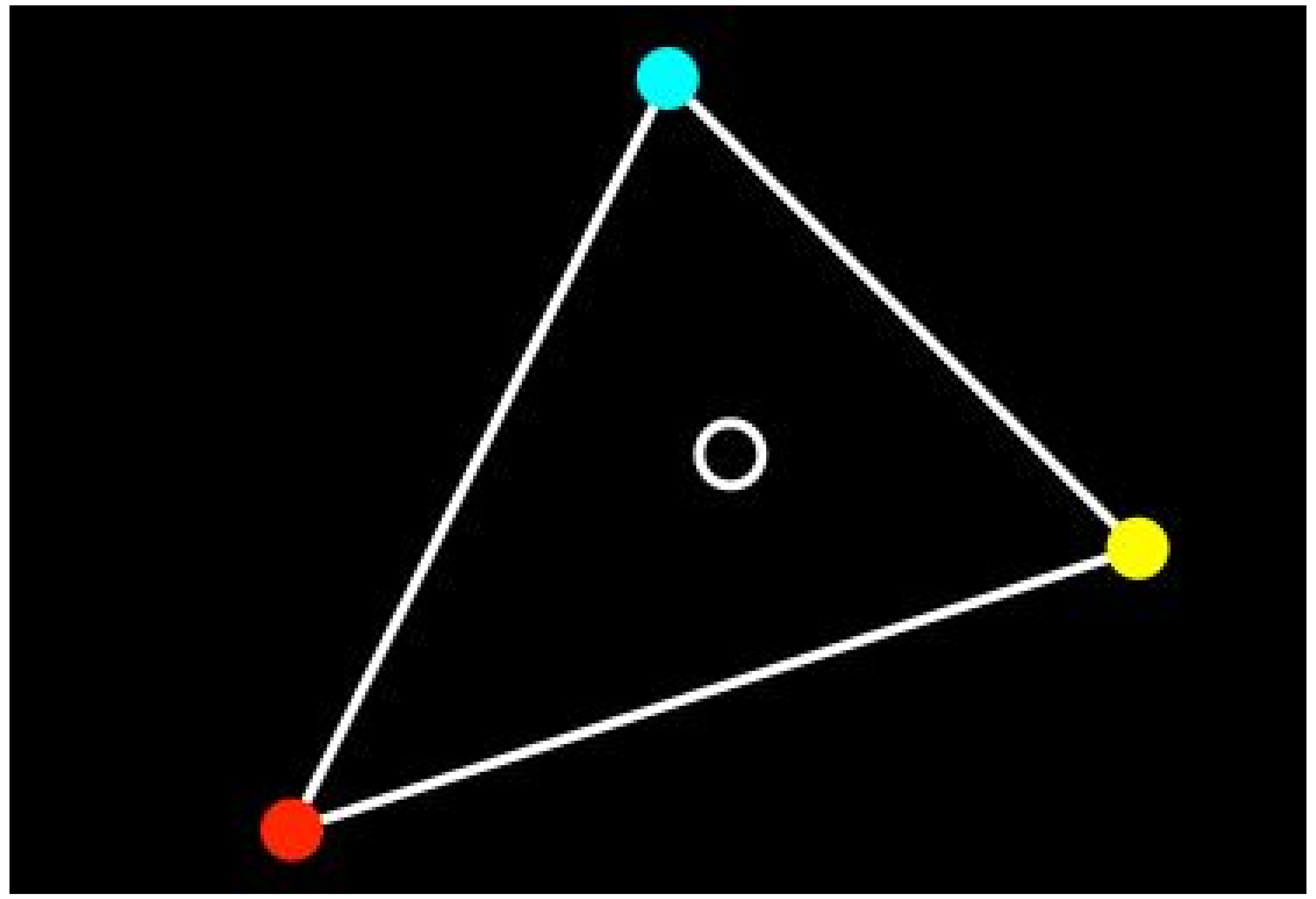
- Use even-odd rule or winding rule
- Easier if polygon is in 2D (project from 3D to 2D)
- Easier for triangles (tesselate polygons)

Point-in-triangle testing

- Critical for polygonal models
- Project the triangle, and point of plane intersection, onto one of the planes x=0, y = 0 or z = 0 (pick a plane not perpendicular to triangle - such a choice always exists)
- Then, do the 2D test in the plane, by computing barycentric coordinates (follows next)

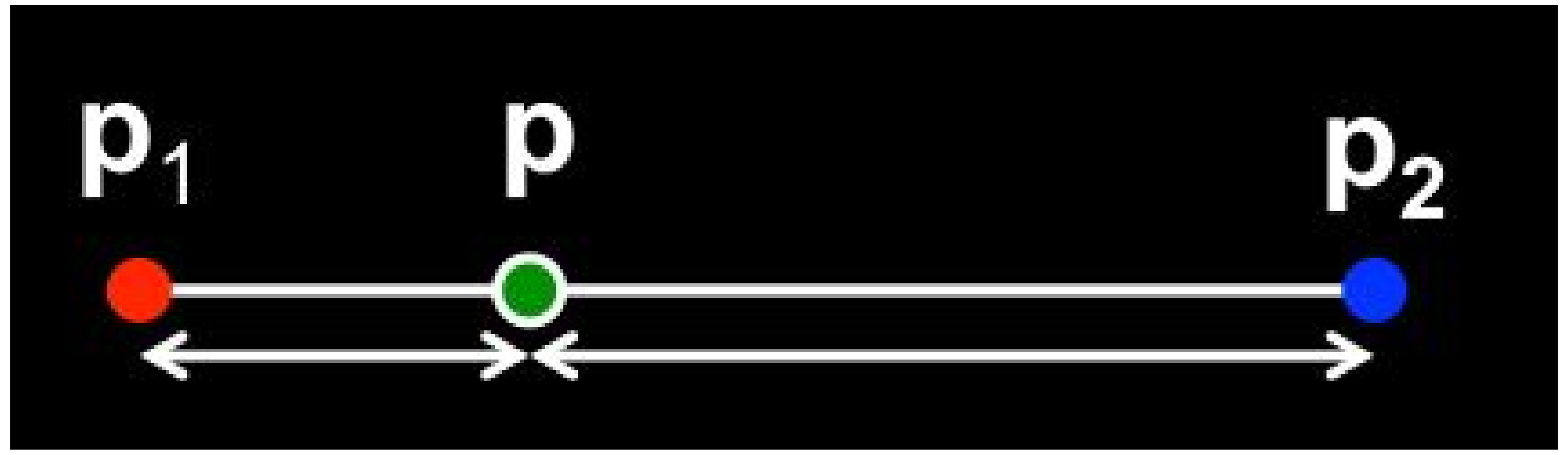
Interpolated Shading for Ray Tracing

- Assume we know normals at vertices
- How do we compute normal of interior point?
- Need linear interpolation between 3 points
- Barycentric coordinates
- Yields same answer as scan conversion



Barycentric Coordinates in 1D

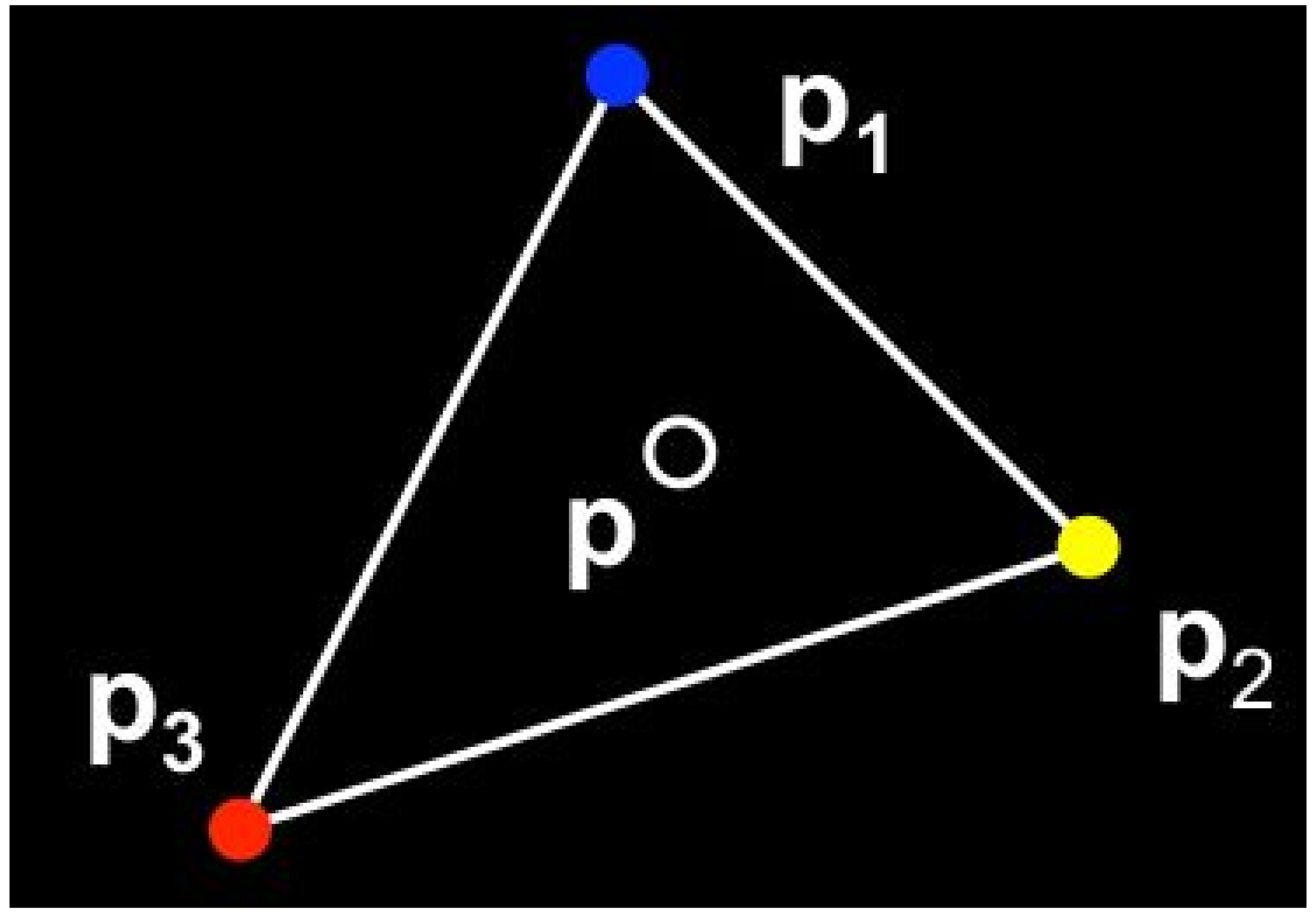
- Linear interpolation
  - $p(t) = (1-t)p_1 + tp_2$, $0 <= t <= 1$
  - $p(t) = ap_1 + bp_2$ where $a+b = 1$
  - p is between p_1 and p_2 iff $0 <= a,b <= 1$
- Geometric intuition
  - Weigh each vertex by ratio distances from ends



  - a,b are called barycentric coordinates

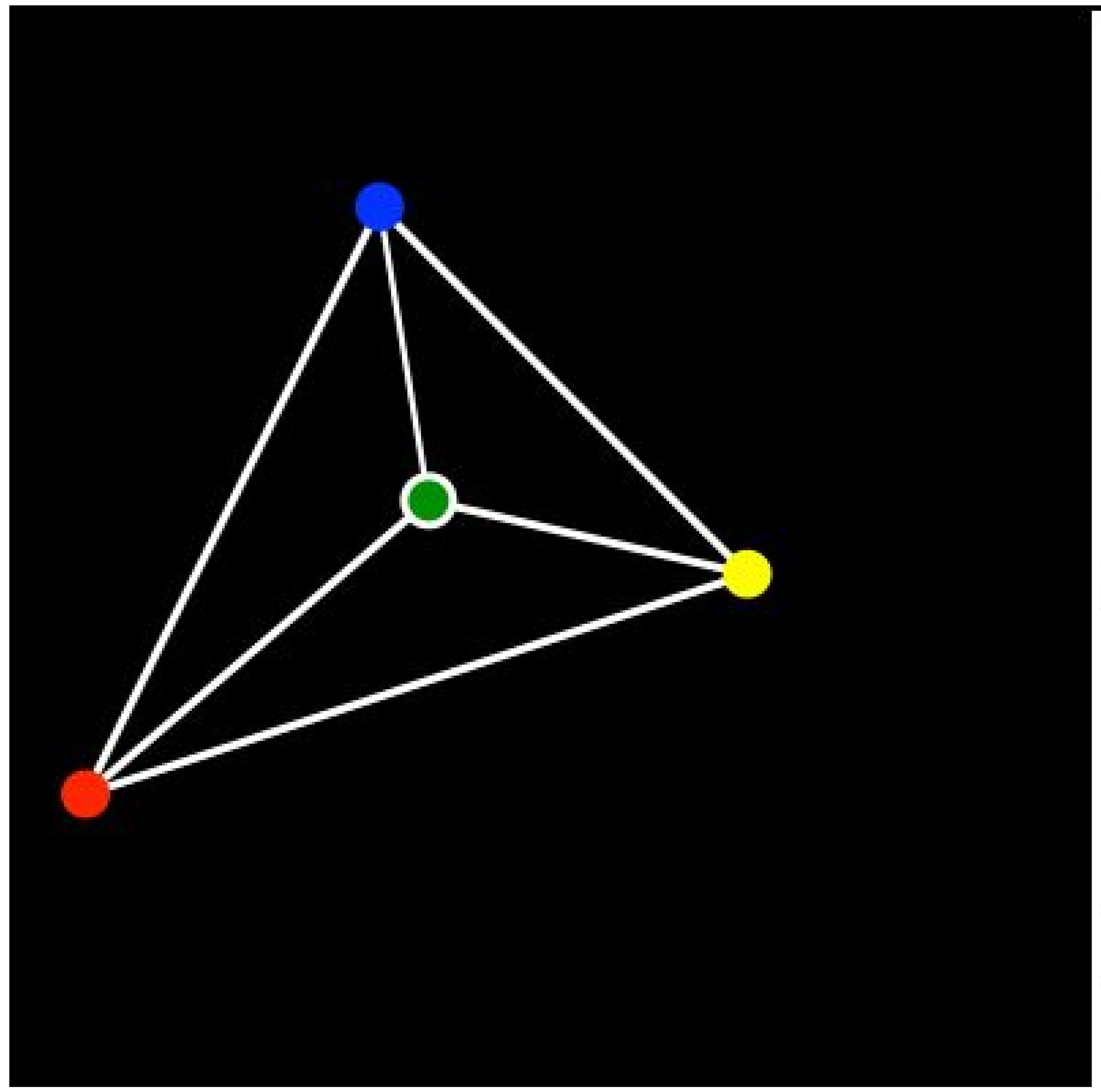
Barycentric Coordinates in 2D

- Now, we have 3 points instead of 2
- Define 3 barycentric coordinates a,b,c
- $p = ap_1 + bp_2 + cp_3$
- p inside triangle iff 0 <= a,b,c <= 1, a+b+c=1
- How do we calculate a, b, c given p?



Barycentric Coordinates for Triangle

- Coordinates are ratios of triangle areas
- Areas in these formulas should be signed, depending on clockwise (-) or anti-clockwise orientation (+) of the triangle! Very important for point-in-triangle test

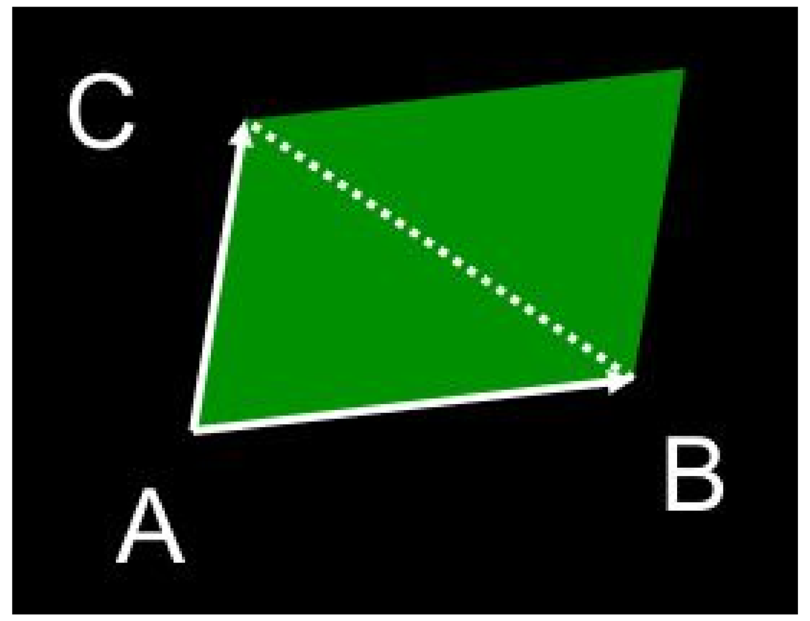


$$\alpha = \frac{Area(\mathbf{C}\mathbf{C}_1\mathbf{C}_2)}{Area(\mathbf{C}_0\mathbf{C}_1\mathbf{C}_2)}$$

$$\beta = \frac{Area(\mathbf{C}_0\mathbf{C}\mathbf{C}_2)}{Area(\mathbf{C}_0\mathbf{C}_1\mathbf{C}_2)}$$

$$\gamma = \frac{Area(\mathbf{C}_0\mathbf{C}_1\mathbf{C})}{Area(\mathbf{C}_0\mathbf{C}_1\mathbf{C}_2)} = 1 - \alpha - \beta$$

Computing Triangle Area in 3D



- Use cross product
- Parallelogram formula
- Area(ABC) = ½ | (B-A)x(C-A)|
- How to get correct sign for barycentric coordinates?
  - tricky, but possible:
    - compare directions of vectors (B-A)x(C-A), for triangles CC_1C_2 vs $C_0C_1C_2$, etc [either 0 (sign+) or 180 deg (sign-) angle]
  - easier alternative: project to 2D, use 2D formula
  - projection to 2D preserves barycentric coordinates

Computing Triangle Area in 2D

- Suppose we project the triangle to XY plane
- Area (xy-projection(ABC)) = $(\frac{1}{2})((b_x-a_x)(c_y-a_y)-(c_x-a_x)(b_y-a_y))$
- This formula gives correct sign (important for barycentric coordinates)

Review

- Ray Tracing/Geometric Queries

Next Lecture

- Spatial Data Structures

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